

# Nonlinear System Identification of Aircraft Dynamics using GLO-MAP

John Valasek, Han Hsun Lu, Monika Marwaha, Puneet Singla

Texas A&M University/ University at Buffalo



AEROSPACE ENGINEERING  
TEXAS A&M UNIVERSITY

Vehicle Systems & Control Laboratory

## Abstract

We introduce the Global-Local Mapping Approximation algorithm as a candidate for identifying nonlinear, six degree-of-freedom rigid body aircraft dynamics. The technique models the nonlinear dynamical model as a sum of a linear model and a nonlinear model. The linear model dynamics are assumed to be perturbed by a nonlinear term which represents the system nonlinearities that are not captured by the linear model. Lyapunov stability analysis is used to derive the learning laws. To demonstrate the suitability of the algorithm for nonlinear system identification of aircraft dynamics, a longitudinal and a lateral/directional example using flight test data is conducted. The true nonlinear model is generated using both the six degree-of-freedom nonlinear equations of motion of an aircraft, and by flight test data.

## Motivation

- Linear approximation is limited to the often small region about the operating point .
- The major drawback of all existing nonlinear system identification algorithm is that they treat the unknown system as a complete Black box.
- To provide the flexibility to vary the order of approximation depending upon the operating point and incorporate any a-priori information about the unknown system dynamics.

## Bibliography

- P. Singla, T. Henderson, J. L. J. and Hurtado, J. E., "A Robust Nonlinear System Identification Algorithm using Orthogonal Polynomial Network," 15th AAS/AIAA Space Flight Mechanics Conference, Copper Mountain, Colorado, 23-27 January 2005.
- Singla, P. and Junkins, J. L., Multi-Resolution Methods for Modeling and Control of Dynamical Systems, CRC Press, Aug. 2008.
- J.-N. Juang, Applied System Identification. Prentice Hall, 1994.
- F. Arthurs, J. Valasek, and M. D. Zeiger, "Precision onboard small sensor system for unmanned air vehicle testing and control," ser. AIAA Paper 2016-1138. San Diego, CA: AIAA Guidance, Navigation, and Control Conference, Jan. 2016.

## Methods

The proposed nonlinear system identification algorithm is that the overall procedure can be split into two sequential sub-processes: linear system identification followed by the nonlinear system identification process. By electing the linear time-invariant model as the starting point, the nonlinear term is added to account for nonlinear departure from the best known linear model.

$$\begin{aligned}\dot{\hat{x}} &= A_l \hat{x} + B_l u + g(\hat{x}) \\ \hat{y} &= C_l \hat{x} + D_l u\end{aligned}$$

Here  $g(\hat{x})$  is a vector of unknown nonlinearities not captured by a-priori knowledge of the system. GLO-MAP blends locally independent approximations to a piecewise globally continuous function. With  $X = \{x_1, \dots, x_N\}$  as a uniform grid with spacing  $h$  and  $F = \{f_1(x), \dots, f_N(x)\}$  as a set of continuous functions that approximates global function  $f(x)$  at points  $x_i \in X$ . We define the weighted average approximation centered on  $i^{th}$  vertex  $x_i$  valid for over  $x \in \{x_i, x_{i+1}\}$ :

$$\begin{aligned}\bar{f}_i(x) &= \omega(\bar{x}_i) f_i(x) + \omega(\bar{x}_{i+1}) f_{i+1}(x), \quad \bar{x}_i \triangleq \frac{x - x_i}{h} \\ f(x) &= \sum_{i=1}^N \omega(\bar{x}_i) f_i(x), \quad \bar{x}_i \in [-1, 1]\end{aligned}$$

Assuming weighing functions to be polynomial, the generic expression for  $m^{th}$  order continuity weight function can be written as

$$\omega(\bar{x}) = 1 - \eta^{m+1} \left\{ \frac{(2m+1)!(-1)^m}{(m!)^2} \sum_{k=0}^m \frac{(-1)^k}{2m-k-1} \binom{m}{k} \eta^{m-k} \right\}$$

Introducing basis functions orthogonal with respect to the weighting function

$$\Phi_n = \frac{1}{c_n} \left[ x^n - \sum_{j=0}^{n-1} \frac{\langle x^n, \Phi_j(x) \rangle}{\langle \Phi_j(x), \Phi_j(x) \rangle} \Phi_j(x) \right]$$

The unknown  $g(x)$  can be expressed as

$$g(x) = \sum_{i=1}^N \omega_i(x) C_i \phi_i(x) = C \Phi(x) \omega(x)$$

Where  $\phi_i(\cdot)$  is a  $N \times 1$  vector of independent basis function set,  $C_i \in \mathbb{R}^{n \times N}$  is a matrix of corresponding basis function coefficients, and  $C \in \mathbb{R}^{n \times N}$  is the matrix of local basis function coefficients for all local approximations.

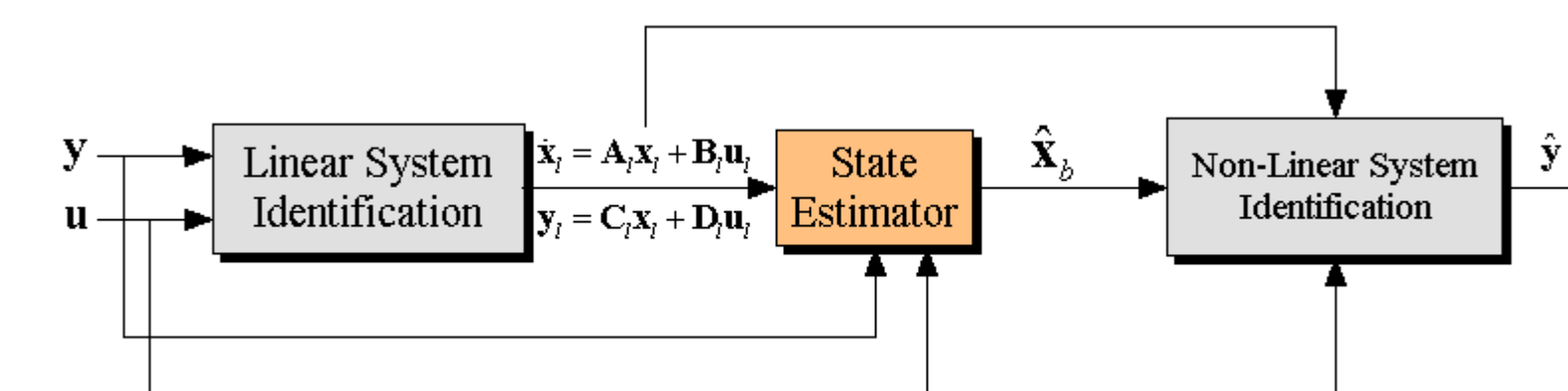


Figure 1. System identification architecture.

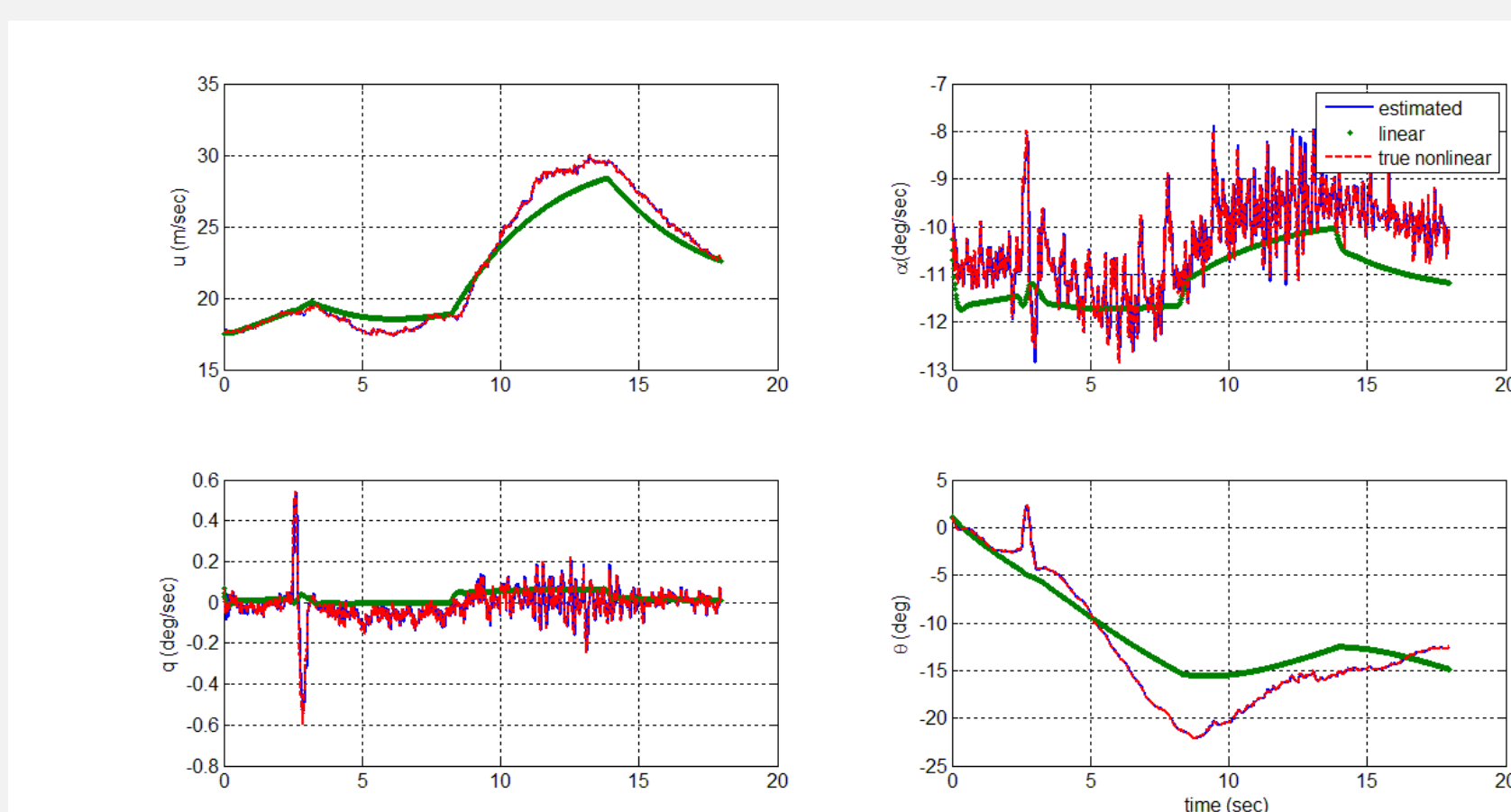


Figure 2. Comparison between GLO-MAP, OKID, and actual flight test results for longitudinal dynamics.

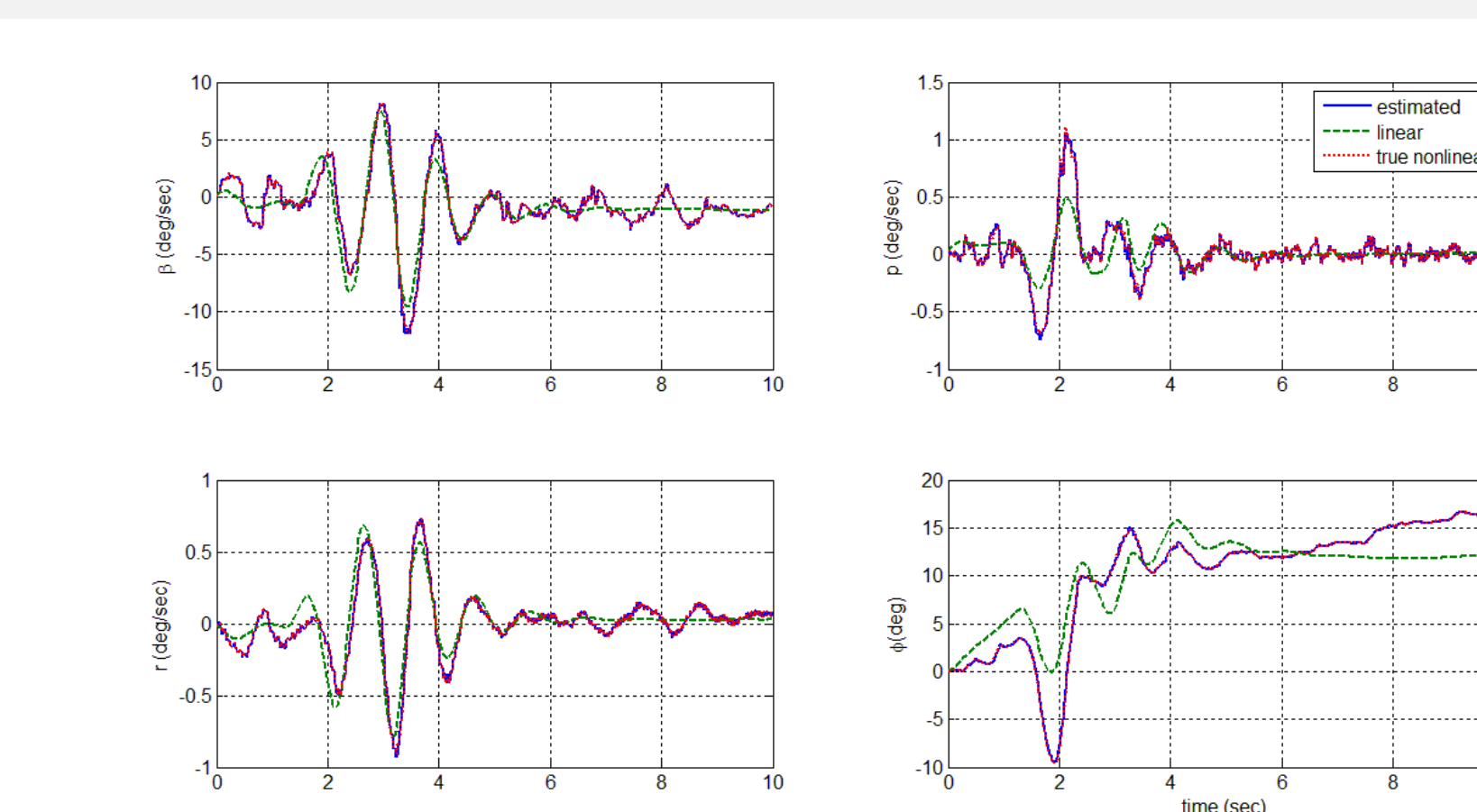
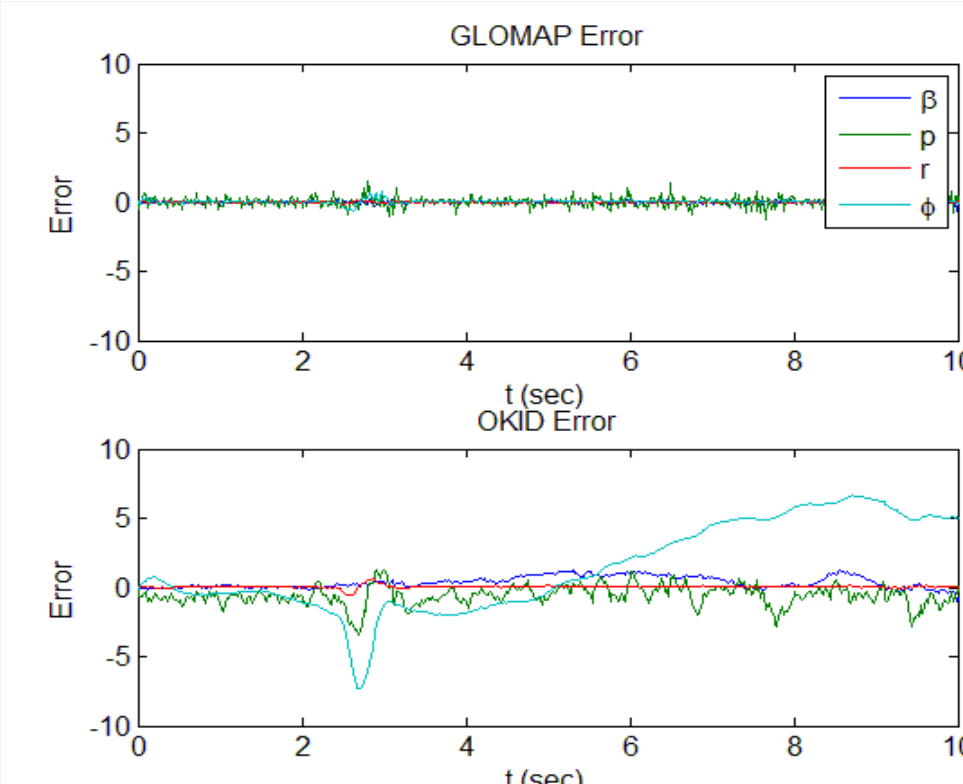
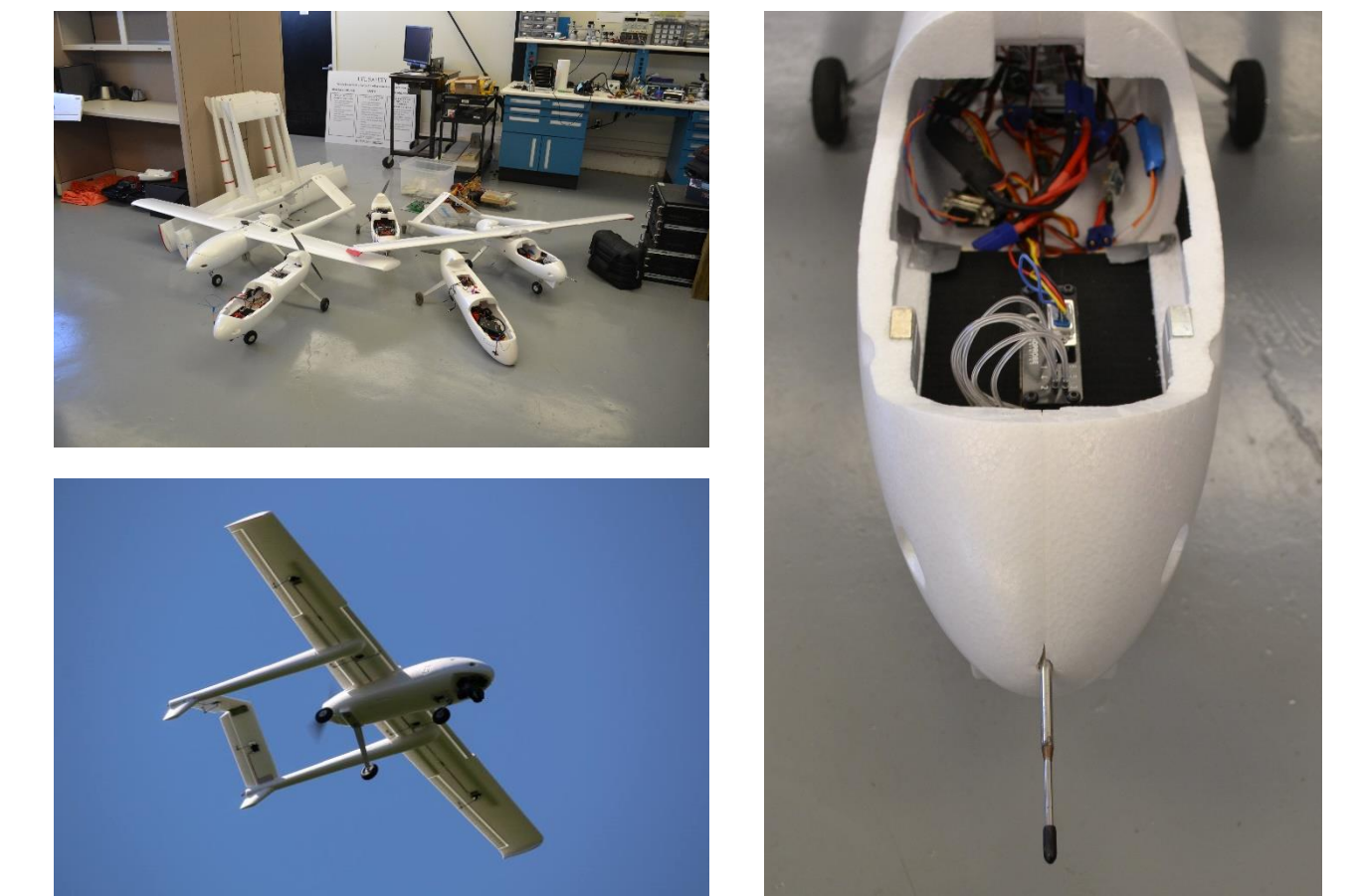
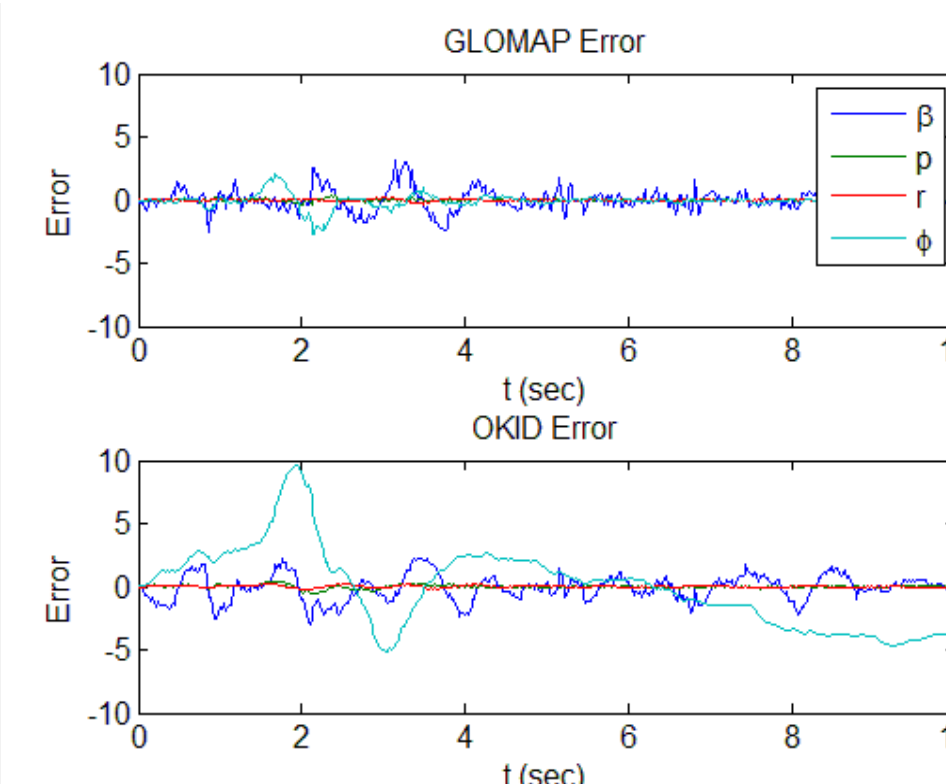


Figure 3. Comparison between GLO-MAP, OKID, and actual flight test results for lateral/directional dynamics.



The Ready Made RC Anaconda is used for flight data collection. The external IMU used is the Vectornav VN-200 supplying attitude angles and angular rates at 100 Hz and GPS information at 5Hz via serial TTL. The Aeroprobe five-hole probe and  $\mu$ ADC are installed to collect the velocity  $u$ , AOA, and  $\beta$ .

Table 1 UAV Characteristics

Wingspan	6.76 ft
Length	4.63 ft
Motor	35-48 800-1000kv brushless outrunner

## Conclusions

Developed and applied the Global-Local Mapping Approximation algorithm to identify the nonlinear dynamics of a rigid body aircraft. Recorded flight data for an small scale UAV were used, and both longitudinal and lateral/directional dynamics were considered. Global-Local Mapping Approximation works well on flight data which has sensor noise and atmospheric disturbances, and is therefore a promising candidate for nonlinear system identification of aircraft dynamics.